

## Estimates of the Steady State Growth Rates for Iran Economy in Solow Growth Model Framework

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### Abstract

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This paper estimates the steady state growth rate for Iran economy. We shall use an extended version of the Solow (1956) growth model, in which total factor productivity is assumed to be a function of human capital (measured by average years of education), and trade openness. Using this framework we show that the education and trade openness have played an important role to improve the long-run growth rate. Our empirical results, with data from Iran, show that trade openness and education have significant and permanent growth effects and a few broad policies to improve these steady state growth rate are suggested.

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**Keywords:** Trade Openness, Steady State Growth Rate, Solow Growth Model, Education

**Jel classification:** C22,C32,O40

### Introduction

In the Solow (1956) growth model the steady state rate of growth of output per worker (*SSGR*) equals to the exogenously determined rate of growth of total factor productivity (*TFP*).

Therefore, this model is known as the exogenous growth model. It is hard to use it to develop policies for growth because the determinants of *TFP* are not known. In contrast endogenous growth literature identifies more than 80 variables as potential determinants of *TFP*; see Hoover and Partez (2004).

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for a survey There are numerous theoretical and empirical studies on the determinants of growth. Theoretical studies are classified into exogenous growth models and endogenous or new growth models. Empirical studies use either cross-section or time series techniques to estimate these theoretical models. Therefore, from an empirical perspective, there are three types of studies on growth. Firstly, cross-section studies based on the endogenous growth theories are the most prolific variety. Secondly, time series empirical works, based on the exogenous growth theory of Solow (1956) are the second most prolific type.

However, many such time series studies give the wrong impression that their specifications are based on the endogenous growth theory. In fact these time series studies use the Solow model without an adequate awareness of its essence. In the Solow model what actually estimated with time series data are the long run Cobb-Douglas production functions and not the long run growth equations. This is so because in the Solow model the long run growth rate is determined by the rate of growth of technological progress (TFP) and its determinants are not known. Thirdly, cross section studies based on the exogenous growth theory are relatively few. The well-known works of Mankiw, Romer and Weil (1992) and its critiques belong to this category.

Time series studies based on the endogenous growth theory are of four types viz., (a) Jones' (1995) calibration techniques to test the predictions of the endogenous growth model, (b) Similarly Kocherlakota and Mu Yi's (1996) use the VAR framework to test the predictions of the endogenous growth models, (c) Greiner, Semler and Gong's (2004) pioneering attempt to estimate the structural parameters of endogenous growth models with time series data and (d) several time series works in which the production function is augmented in an *ad hoc* manner with shift variables like human capital, openness of trade, aid, foreign direct investment and infrastructure expenditure etc. However, it is not clear from this last category whether the estimated long run equation actually is a production function or a growth equation although such studies incorrectly claim that it is the latter. This is important because cointegration techniques are used to estimate only the implied long run relationships in the levels of the variables and not in their growth rates.

While the econometric techniques of these three approaches are satisfactory, they seem to have specification weaknesses because it is hard to accept that annual growth rates of output or even average growth rates over 3 to 5 years adequately measure the dependent variable viz., *SSGR*.

This is so because simulations with the closed form solutions show that an economy takes several periods to converge to anywhere close to its steady state. This transition period may be as long as 25 to 30 years even for small perturbations. Baldwin (2004), Dollar and Kraay (2004), Edwards (1992) and Winters (2004), are among a few who explicitly note that the transition period from one to another steady state may span over two or three decades. Therefore, while the dependent variable in the cross section studies viz., average growth rates of 20 or more years is a good approximation to the steady state growth of output, it is hard to accept that the dependent variable is a good measure of the *SSGR* in the panel and annual time series studies. In this paper we show how to estimate the growth effects of a growth enhancing variable with country specific annual data with an extended Solow model. We investigate this aspect with an extended version of the Solow (1956) growth model by incorporating education and trade openness as key determinants of the long-run growth rate.

### Specification of the Model

In the Solow (1956) growth model the long run equilibrium growth of output (in per worker terms) is determined by the rate of technical progress (TFP). However, the determinants of TFP are not known although its contribution to growth is as much as 50% in some advanced economies. The Solow (1956) growth model, therefore, is known as the exogenous growth model. TFP is usually estimated as a residual from the growth accounting framework of Solow (1957) and also known as the Solow residual (SR). In our view SR is more like a measure of our ignorance of the determinants of growth rather than an estimate of the true TFP. An important feature of Solow (1956) model is its final conclusion that, in the long run, per worker income grows only at the rate at which TFP grows ( $g$ ) and an increase in the investment ratio (ratio of investment to output) has no long run growth effects. Extensions to the Solow model, such as Mankiw, Romer and Weil (1992), MRW hereafter, essentially aim to reduce the size of the SR or our ignorance about the determinants of growth.

The starting point is the steady state solution for the level of output in the Solow (1956) growth model and this is:

$$Y^* = \left( \frac{s}{n+g+d} \right)^{\frac{\alpha}{1-\alpha}} \cdot (A) \quad (1)$$

where  $y^*$  ( $Y / L$ ) is the steady state level of income per worker,  $s$  = the ratio of investment to income,  $d$  = depreciation rate of capital,  $g$  = the rate of technical progress,  $n$  = the rate of growth of labour,  $A$  the stock of knowledge and  $\alpha$  the exponent of capital in the Cobb-Douglas production function with constant returns (see below). This implies that the steady state rate of growth of per worker output (SSGR), assuming that all other ratios and parameters are constant, is simply TFP because:

$$\ln y \approx \text{SSGR} \approx \ln A \approx \text{TFP} \quad (2)$$

However, since the determinants of TFP are not known and are exogenous to the Solow (1956) growth model, the Solow model is also known as the exogenous growth model. The new growth theories based on Endogenous growth models (ENGMs) use optimization framework and suggest several potential determinants of TFP. We extend the Solow model as follows. Note that the SSGR can be estimated by estimating the production function. The production function can also be extended by assuming that the stock of knowledge ( $A$ ) depends on some important variables identified by the ENGMs. We start with the well-known Cobb-Douglas production function with constant returns:

$$Y_t = A_t K^\alpha L_t^{1-\alpha} \quad (3)$$

Following Rao (2010) and Paradiso and Rao (2011) we assume the following general evolution for the stock of knowledge  $A$ , where for simplicity the vector  $Z$  may consist of more than one variable, whereas  $S$  and  $W$  are assumed to consist of one variable each.

$$A_t = A_0 e^{(\gamma_i Z_{it} \cdot T + \partial_1 S_t + \partial_2 S_t^2 + \eta W_t)} \quad (4)$$

Transforming (3) into its intensive form, substituting (4), and taking the logs we have:

$$\ln y_t = \ln A_0 + \gamma_i Z_{it} \cdot T + \partial_1 S_t + \partial_2 S_t^2 + \eta W_t + \alpha \ln k_t \quad (5)$$

where  $y = (Y / L)$  and  $k = (K / L)$ . Equation (5) captures the actual level of output due to two types of variables viz., factor accumulation and due to variables other than factor accumulation such as  $Z$ ,  $S$  and  $W$ . Specification of these other variables may affect output is an empirical issue. Their effects may be trended ( $Z$ ) or nonlinear ( $S$ ) or simply linear ( $W$ ).

In the steady state, when  $\Delta \ln k \rightarrow 0$ , the steady state growth rate (SSGR) is equal to the growth rate of the stock of knowledge ( $\Delta \ln A$ ). There are two ways to measure the SSGR. One restrictive method considers all the changes in the variables as zero; whereas a less restrictive one considers only  $\Delta \ln k = 0$ . We call the first SSGR as *SSGR1*, and the second as *SSGR2* and are as follows. *SSGR1* and *SSGR2* can also be interpreted as the medium run and long run estimates of the growth rate and they are:

$$SSGR_1 = \partial_i Z_{it} \quad (6)$$

$$SSGR_2 = \gamma_i \Delta Z_{it} \times T + \gamma_i Z_{it} + \partial_1 \Delta S_t + 2 \partial_2 \Delta S_t \times S_t + \eta \Delta W \quad (7)$$

We make use of both of these measures of SSGR and try to understand the potential factors influencing the SSGR and how policy can improve them.

### Regression Estimation Techniques

This study estimated long run elasticities with three methods namely Fully Modified Ordinary Least square (FMOLS) of Phillip and Hansen (1990), Canonical Co-integration Regression (CCR) of Park (1992) and Dynamic Ordinary Least square (DOLS) developed by Stock and Watson (1993). This study adopted these regression techniques as the variables found co-integrated.

Firstly, study employed FMOLS; this technique allows a semi parametric correction for auto correlation in co-integrating vectors and resolve endogeneity issue.

To avoid the issue caused by long run correlation between stochastic regressors and co-integrating equations, it used covariance matrices of residuals. Secondly, study employed Canonical Co-integrating Regression (CCR) technique that permits asymptotic Chi2 testing.

$$8) \ln y_t = Intercept + \gamma_1 EDU_t.T + \partial_1 EDU_t + \partial_2 EDU^2 + \alpha \ln k$$

This technique used transformed data that involves simple adjustments of integrated processes using stationary components in co-integrating models. Thirdly, this study employed Dynamic Ordinary Least Square (DOLS) technique that adds lags and leads of first differenced regressors to specifications. Our selected growth-enhancing variables are: trade openness (OPEN), and human capital index, measured as average years of education (EDU). Definitions of variables and sources of data are in the appendix. We expect that all variables enter in the estimation of iran economy but multi-collinearity problem between variables may arise and some of these variables could be not statistically significant. In the paper we report only the estimations showing plausible, economically and statistically results.

three models are compatible with iran:

$$9) \ln y_t = \text{Intercept} + \gamma_1 \ln OPEN_t \cdot T + \partial_1 EDU_t + \partial_2 EDU^2 + \alpha \ln k$$

$$10) \ln y_t = \text{Intercept} + \gamma_1 EDU_t \cdot T + \gamma_2 \ln OPEN_t + \partial_1 EDU_t + \partial_2 EDU^2 + \alpha \ln k$$

In the first model, shown in Table 1, equation (10) is estimated with  $Z_{1t}=EDU_t=S_t$  And in this model  $Z_{1t}=\ln OPEN_t$  and  $S_t=EDU_t$ . According to equation(8), EDU has two components: one non-linear component and one long-run component multiplied for the trend. This is because education may have non-linear effects as discussed by Rao et. Al (2010). The estimation results of equation (8) are reported in Table 1. The estimate of equation(8) is satisfactory in that all of its coefficients are correctly signed and statistically significant. The EG residual test shows that a cointegration exists at 5% level of statistical significance. The ECM shows a factor loading ( $\lambda$ ) significance and with the expected negative sign. The diagnostic tests show that the model is correctly specified.

**Table (1). Results of equation (8)**

<b>InK</b>	<b>0.179</b> <b>(0.048)</b>	<b>0.148</b> <b>(0.223)</b>	<b>0.203</b> <b>(0.047)</b>
$\lambda$	<b>-0.63</b>		
<b>EG residual test</b>	<b>-1.313</b>		
<b>LM(1) test (p-value)</b>	<b>0.443</b>		
<b>LM(2) test (p-value)</b>	<b>0.239</b>		
<b>LM(4) test (p-value)</b>	<b>0.423</b>		
<b>JB test (p-value)</b>	<b>0.201</b>		
<b>BPG test (p-value)</b>	<b>0.443</b>		

Note: FMOLS= fully modified ordinary least squares; CCR = canonical cointegrating regression; DOLS = dynamic ordinary least squares; GETS = general to specific; and EG = Engle-Granger t-test for cointegration.  $\lambda$ , factor loading in the ECM. BPG = Breusch-Pagan-Godfrey heteroskedasticity test; JB = Jarque-Bera normality test; LM = Breusch-Godfrey serial correlation LM test. FMOLS uses Newey-West automatic bandwidth selection in computing the long-run variance matrix. In the DOLS leads and lags are selected using the AIC criteria. The standard errors (not reported) for the DOLS estimation are calculated using the Newey-West correction.

These formulations for growth rate are based on empirical considerations and in our case specification gave the best empirical results. In equation (11),  $\sigma_1$  measures SSGR due to  $EDU_t$ . The SSGR effects of  $EDU_t$  are assumed to have some dynamic component in model (8), which are captured by  $EDU$  and  $EDU^2$ . In the long run, however, all the differences of the variables become zero in the steady state. Therefore, the SSGR is:

$$11) \quad SSGR = \sigma_1 EDU_t$$

we estimated model (9) in which  $EDU$  is replaced with  $\ln OPEN$  as the determinant of SSGR, that is  $Z_{1t} = \ln OPEN_t$  and  $S_t = EDU_t$ . The results are in Table 2 and it can be seen that all estimates are significant.

**Table (2). Results of Equation (9)**

	<b>FMOLS</b>	<b>DOL</b>	<b>CCR</b>
		<b>S</b>	
Intercept	20.893 (0.0000)	20.800 (0.000)	20.892 (0.000)
EDU	0.100 (0.010)	0.096 (0.052)	0.103 (0.005)
EDU^2	0.016 (0.006)	0.016 (0.031)	0.017 (0.005)
lnOPEN.T	0.007 (0.000)	0.007 (0.000)	0.007 (0.000)
lnK	0.191 (0.034)	0.163 (0.197)	0.234 (0.030)
$\lambda$	-0.64		
EG residual test	-1.044		
LM(1) test (p-value)	0.529		
LM(2) test (p-value)	1.451		
LM(4) test (p-value)	0.799		
JB test (p-value)	0.303		
BPG test (p-value)	0.850		

The speed of adjustment ( $\lambda$ ) implies negative feedback mechanism and is statistically significant at 1% level. The Engle and Granger (1987) t-test supports the existence of cointegration among the variables at 1% level. Moreover, the diagnostic tests indicate no issues with respect to serial correlation, normality and heteroscedasticity. Aside from using the average year of total schooling as a measure of human capital, we also utilized alternative measures such as the total school enrolment rate, average year of primary schooling, average year of secondary schooling and average year of tertiary schooling to determine the SSGR.

In equation(10) we also include both EDU and lnOPEN as determinants of SSGR. In this case  $S_t = EDU_t$ ,  $Z_{1t} = EDU_t$ ,  $Z_{2t} = \ln OPEN_t$ . Note that there is an additional term for education ( $EDU \times T$ ) compared to the specification in model 9. This implies that some of the non-linear effects of EDU are offset by the underlying trend in EDU, which may be due to improvements in the quality of education over time. The results are in Table 3 and are impressive.



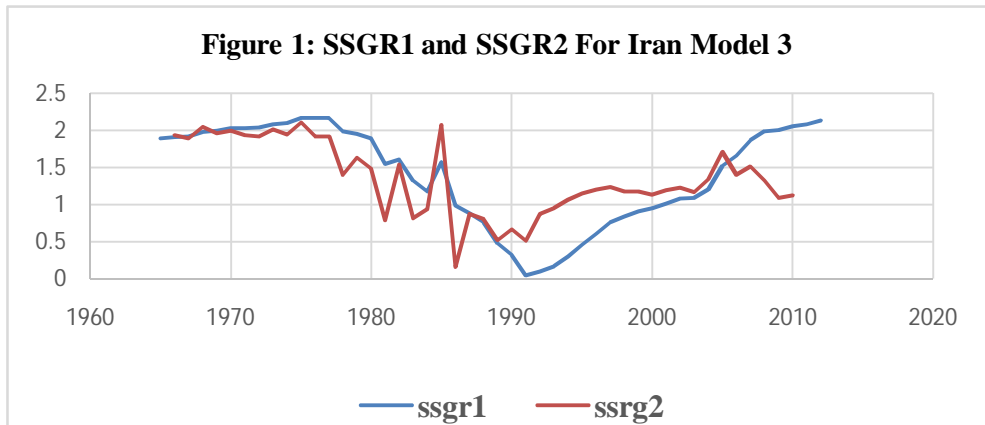
All the coefficients are statistically significant, the coefficient of capital is close to one third, the residual tests (EG) confirm the existence of a long-run relationship and ECM is satisfactory. This is our preferred estimate and we use this model to compute the dynamics of SSGR in equations (6) and (7).

**Table (3): Results of Model (10)**

	<b>FMOLS</b>	<b>DOLS</b>	<b>CCR</b>
Intercept	20.518 (0.0000)	22.547 (0.000)	20.892 (0.000)
EDU	0.284 (0.000)	0.330 (0.076)	0.148 (0.001)
EDU^2	0.020 (0.005)	0.031 (0.205)	0.023 (0.002)
EDU.T	0.007 (0.000)	0.006 (0.000)	0.004 (0.000)
lnK	0.255 (0.000)	0.114 (0.443)	0.203 (0.047)
lnOPEN	0.003 (0.389)	0.001 (0.854)	0.006 (0.432)
EG residual test	-1.161		
LM(1) test (p-value)	1.823		
LM(2) test (p-value)	1.669		
LM(4) test (p-value)	1.011		
$\lambda$	-0.62		
BPG test (p-value)	0.553		

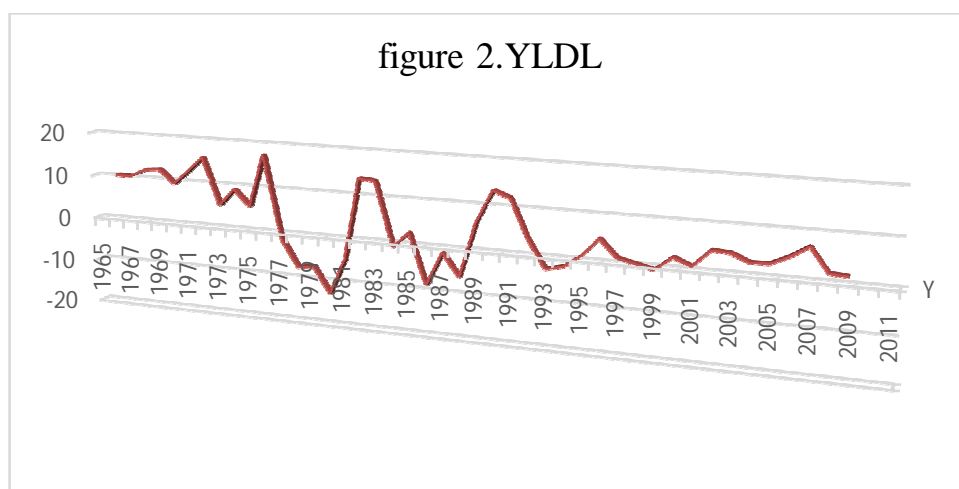
The growth effect of EDU is 0.284 and statistically significant at the conventional levels. In, CCR and FMOLS the capital share is between 0.2 to 0.4, however the DOLS technique produced implausibly low estimate at around 0.11. Further the estimates of capital share are statistically significant at conventional levels in FMOLS and CCR technique. but coefficient of trade openness is not significant.

The contributions of TRADE and EDU to SSGR2 and EDU to SSGR2 are shown in Figure 1. SSGR2 shows the most dynamic pattern and is closely linked to the actual growth rate of per worker GDP.



Note:  $SSGR = 0.284 * EDU$

Since all techniques yield consistent results, we are confident that our model is correctly specified. The estimate of growth effects of EDU is 0.284 and hence we use this value to compute the dynamics of SSGR (see equation 11). The plot of SSGR1 and SSGR2 presented in Figure 1. The average value of SSGR is around 1.35% over the period 1965 to 2012. In SSGR2 the contribution of EDU with OPEN to the long run growth rate has been decreasing since 1978, Reduce the growth in Iran refer to war years, between (1982-1990), therefore since 1990, SSGR1 and SSGR2 are increasing. The declining trend in Iran's SSGR2, seems to be due to two reasons. Firstly, as stated above, trade openness may not have played an effective role in the early stages of its development and protectionist pressures may have sheltered some inefficient domestic industries and this has increased in Iran from a 1.15% during the 1990s and 2000s to 2.2% by 2012. Increase SSGR since 1990 due to two options. First, Iran may increase its absorption of efficient technologies and management practices from the advanced countries management practices from the advanced countries, second, increase in export and import after war, and consequently, improvement of the foreign trade, in SSGR<sub>2</sub>, actual growth of output per worker (DLYL) presented in Figure 2.



The average rate of growth of output per worker during 1965-1990 and 1990-2012 are, respectively, 1.55% and 2.51%, implying that currently Iran is not far from its  $SSGR_1$  of 1.4% and  $SSGR_2$  1.35%. The plot of the actual rate of growth of per worker output (computed with the actual values of GDP and employment growth) is shown in Figure 2. There is a mild upward trend of 0.007 in the SSGR which is encouraging. Due to short sample (1965-2012), we used only FMOLS technique although all techniques yield consistent results. Yet this country seems to be growing below its steady state growth rate in some years. Such a low steady state growth rate may be due to some negative externalities, especially due to the political instability and appropriate management in Iran. Therefore, we cannot claim that our results for the Iran have adequately captured all the relevant externalities. Further work is necessary to draw definitive conclusions but it may be said that increased trade liberalization may make the coefficient of TRADE significant. But from 2004 onward the impact of economic sanctions on Iran economy, can not be ignored.

## Conclusions

This paper used an extended Solow (1956) growth model to estimate the long run growth rate for Iran for the period 1965-2012. Three time series techniques (CCR, FMOLS and DOLS) were utilized to estimate the cointegrating equations. Country specific time series studies are important because it is hard to justify the basic assumptions of the cross-section.

We then derive the implications from the estimated parameters of the production function for the relationship between the *SSGR* and trade openness and education. All the coefficients are statistically significant and with expected signs, but coefficient of trade openness is not significant in equation (10). In all estimated models, the residual tests (EG) confirm the existence of a long-run relationship and ECM is satisfactory. We have computed the *SSGRs* for Iran economy and showed that the human capital index (*EDU*) and trade openness (*OPEN*) explain much of the dynamics of the *SSGRs* in Iran. The capital share is around 2%, and has become statistically significant, and the average value of  $SSGR_1$  is around 1.40%, and  $SSGR_2$  is 1.35% over the period 1965 to 2012. More importantly, the estimate of growth effect of *EDU* is 0.284 in equation (10). The fact that human capital measured as average year of total schooling has a permanent growth effect in Iran and implies that meaningful advice for policy makers can be drawn. But, Iran needs to liberalize trade to absorb more efficient technologies and management skills and needs to improve its learning by doing (LBD) programmes. However, Iran should be brought to an improvement in external relations and foreign trade to compensate for the negative effects of the crisis and Economic sanctions. We hope that our approach and empirical findings would be useful for further extensions to the growth models framework to develop policies to permanently increase the long-run growth rates in Iran and other developing countries.

### Data Appendix

**Y**= Real GDP; **L**= Employment (Total economy); **EDU**= Human Capital Index measured as average years of education, average year of total schooling as a measure of human capital, alternative measures such as the total school enrolment rate, average year of primary schooling, average year of secondary schooling and average year of tertiary schooling; **OPEN**= Ratio of imports plus exports to GDP. All data, are taken and constructed from **WDI** database and **PWT7** (pen world table) Center for International Comparisons University of Pennsylvania.

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